

# Our String Field Theory Liberating Left and Right Movers as Constituent “Objects”

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## abstract

We review the idea of our earlier proposed string field theory [1–3], which makes the second quantized string theory appear as described by one or two types of stationary - so called - “objects” for string theories respectively with and without open strings. It may be better to look on our string field theory as a *solution* of a second quantized string theory, in which we have decided to ignore, how strings are topologically hanging together. Rather we satisfy ourselves with realizing solely the information contained in the knowledge of, through which points in space time passes a string. In the formulation of the string field theory, in which we have rewritten the systems of strings into a system of what we call “objects”, the scattering of strings take place without any of the “fundamental” “objects” (technically “even objects”) changing. They are only *exchanged* instead. A route to extract from our formalism the vertex of the Veneziano model theory is sketched, and thus in principle we have a line of arguments leading to, that our string field theory indeed gives the Veneziano model scattering amplitudes for the scatterings, that are in fact in our model *only exchanges*.

PACS numbers: 11.25.-w, 11.27.+d, 11.10.-2, 03.70.+k, 11.25.Wx

Keywords: String Field Theory, String Theory, Solvable Models

## I. INTRODUCTION

In our earlier [1, 2] and in a coming work [3] we have presented a string field theory idea, the characteristic feature of which is, that it is based on the observation, that the left and right moving wave patterns on the strings are conserved according to a certain conservation theorem to be explained. Really that means, that there are so many conservation laws, that we may consider our picture a *solution* of string theory. Although to some extent, if our formulation works as we suggest and almost prove, it should represent in some way the same string field theory as the other string field theories on the market [4–12], there is *one important difference* in as far as in our picture some information considered physical in the other models is considered not physical and thus *ignored* in ours! In fact we consider the question of how various pieces of strings are connected to each other as being *not a physical question*. Rather one can in our string field theory only discern the way in which various pieces of strings hang together from the continuity (as function of the string parametrizing parameter  $\sigma$ ) of the position(or momentum) variables. If thus a couple of

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To appear in the Proceedings of the 15th Workshop “What comes beyond the Standard Models”, Bled, July 2012 eds. Norma M. Borstnik, Holger B. Nielsen and D. Lukman.

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strings happen to have a common point, our formulation truly lacks an information compared to the usual [4–12] type of string field theories. From our point of view we might think that the older string field theories [4–12] are complicated, because they *keep* this information about the hanging together of the string pieces. Figure 1 illustrates the type of distinction, which we ignore in our

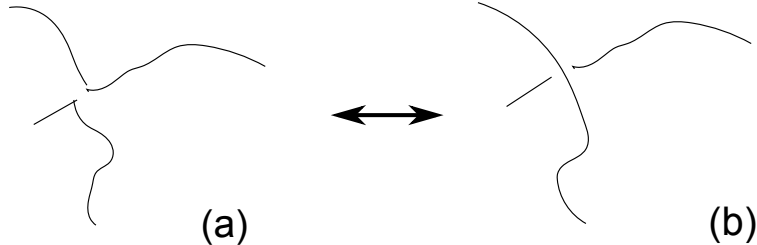


FIG. 1: In the very moment a scattering takes place for here a couple of open strings except that the couple of strings get split and the pieces united in a different way. But wherever there is some string bit it remains on both Figures 1(a) and 1(b).

model.

In that sense the conventional theories seek to keep track of some hanging together information - which we ignore - and there have to have a relative to our theory a complication by having to store some way that - in our point of view - *extra* information in the formalism of the “Fock space”.

But since this information is almost already contained in the continuity, it is only when two strings pass through the *same* point that it gets really important, and since this has zero-probability for happening, we may claim that at the end the important deviation is only on a null set of configurations! But there *is* a difference. And that fact of course ensures that our model is at least *new* in literature.

Also it means that our development of our string field theory is not built on the previous string field theories, but rather takes its outset alone from string theory, almost as going back to its initiation[13–17]. Really we basically start from the single string description and only use string field theory in the sense, that we - in words - have in mind that there are several strings present at the same time.

To make sure that our string field theory, as we shall describe it, in its terms of so called “objects”(which are mathematically related to the strings in a slightly complicated way to be described below or found in our earlier works on this string field theory[1–3]) is indeed essentially (ignoring some null sets) equivalent to usual string theory and thus also to the older string field theories [13–17] we would like to show that it leads to the Veneziano model [22] and the usual string spectrum. The latter we have shown in the coming up work [3], but it is not so trivial again as one might at first think, because we have the single string theory as our outset; the point namely is to show that we out of our “object”-formalism effectively can deduce sufficiently much of the single string theory so as obtain this spectrum.

In the present article it is the major goal to come close to deducing, that our “object”-formalism describing potentially an arbitrary number of strings and thus our “string field theory” leads to the Veneziano model amplitudes, of course via getting rewritten into the derivation of Veneziano amplitudes from single-string or rather a few strings theory. In the next section II we shall review our string field theory by sketching the connection between our “object”-Fock space and string theory with several strings. In section III we illustrate by a slightly oversimplified analogy to an infinitely loosely bound state scattering during which the constituents do not interact, so that the scattering truly gets very formal only. In the following section IV we start developing a correspondence-formalism for a single open string and its corresponding close circular chain of “objects” call  $F$ . In section V we Fourier-transform - in the string  $\sigma$ -variable - the correspondence

between strings and object formulation for an open string. In this Fourier-transformed form we identify the usual creation and annihilation operators acting on the single string states in usual single string formulation. In section VIA we describe the calculational trick of going to a frame in which the plus-component of one of three strings involved in the vertex, which we compute, goes to zero. It is this trick that makes that string couple just to one point formally to the other strings considered otherwise to be just one of them becoming the other one. In the last section VII we conclude, that we have partly checked that new “object”-based “string field theory” represents string theory.

## II. SETTING UP OUR MODEL

We can naturally describe the model of ours in *two* opposite ways/orders, since we can go either one way or the other.:

- **objects**  $\rightarrow$  **strings** That is to say we can first describe the “ontological picture” of our model, which consists of one if the model has “even objects” for which we construct (a) Fock space(s). Next we then describe the mathematical definitions by means of which we rewrite this/these Fock space(s) into a string theory - a string field theory really - so that one can e.g. calculate scattering amplitudes and thereby - we hope - obtain Veneziano amplitudes.
- **strings**  $\rightarrow$  **objects** Alternatively we could start from the idea of having a string field theory in mind in the sense that we like, Kaku and Kikkawa[4] e.g. consider it that we have the possibility of having present any number of strings- even we can have a state being a superposition of states with different numbers of strings present -. Then we rewrite the set of strings present in terms first of the right and left mover fields on the strings  $X_R^\mu(\tau - \sigma)$  and  $X_L^\mu(\tau - \sigma)$  - a set of 26 functions for each string (two sets for a closed string) and next the derivatives of these functions are represented at the end by our “objects”.

### A. The String to “Objects” Way

Let us first describe the second way - from strings to objects - a bit more in detail:

We consider a string field theory state in principle, if we just think of a set of strings in various states. Let us - to be definite- call the number of strings present  $\Xi$  and enumerate a string among these  $\Xi$  ones by ( $\iota$  or  $\kappa = 1, 2, \dots, \Xi$ .) and then in the usual way we split the solution for the single string equations of motion

$$(\partial_\tau^2 - \partial_\sigma^2)X^{\iota\mu}(\tau, \sigma) = 0 \quad (\iota=1,2,3,\dots,\Xi) \quad (1)$$

into right and left movers - at least locally -

$$X^{\iota\mu}(\tau, \sigma) = X_R^{\iota\mu}(\tau - \sigma) + X_L^{\iota\mu}(\tau + \sigma). \quad (2)$$

The splitting into the left  $X_L^{\iota\mu}(\tau - \sigma)$  and the right mover part  $X_R^{\iota\mu}(\tau + \sigma)$  is a bit ambiguous in as far as a constant could be moved from left to right or opposite. This ambiguity, however, disappears, if we instead consider the derivatives with respect to of these left and right mover fields on the strings,  $\dot{X}_R^{\iota\mu}(\tau - \sigma_\iota)$  and  $\dot{X}_R^{\iota\mu}(\tau - \sigma)$ . (Here the dot denotes derivative with respect to say  $\tau_\iota$ ).

Now we shall have in mind that, while for say a closed string the right mover derivative field  $\dot{X}_R^{\iota\mu}(\tau_\iota - \sigma_\iota)$  commute with the left mover derivative field for string  $\iota$  say, then e.g. the right mover

fields derivative for various values of the argument  $\tau_{lR} = \tau_l - \sigma_l$  do *not* commute. In fact we have the commutation relation for these derivatives of the form

$$[\dot{X}_R^{\iota\mu}(\tau_R), \dot{X}_R^{\kappa\nu}(\tau'_R)] = i2\pi\alpha' \delta^{\mu\nu} \delta^{\iota\kappa} \frac{\partial}{\partial \tau_R} \delta(\tau'_R - \tau_R). \quad (3)$$

Now the main idea of our string field theory is to notice that thinking classically the *set* of vectorial values in the 25+1 dimensional Minkowski space of e.g.  $\dot{X}_R^{\iota\mu}(\tau_l - \sigma_l)$  does *not* change as time - let us hereby at least crudely think of  $\tau_l$  as a time - passes by, because of  $\dot{X}_R^{\iota\mu}(\tau_l - \sigma_l)$  only depends on the combinations  $\tau_l - \sigma_l$ . Then namely a change in the “time”  $\tau_l$  can be replaced by a corresponding shift in  $\sigma_l$  and in that way ensure that for some  $\sigma_l$  the same values in the 25+1 dimensional Minkowski that are achieved for one  $\tau_l$ -value will also be achieved for another  $\tau_l$ -value. In that way the set of values taken on remains the same: it is constant in “time”. Because of this fact we can think of the string  $\iota$  as described by its right and left set of mover derivative vectorial values. Well, these say two sets of right and left movers - to take the closed string case - do not 100 % though describe the state of the string  $\iota$ , but it is very close to be so. In fact there is of course an ambiguity with respect to adding a constant to  $\dot{X}_R^{\iota\mu}(\tau_l - \sigma_l)$  and another constant to  $\dot{X}_L^{\iota\mu}(\tau_l - \sigma_l)$ , which in turn would leave an ambiguity of adding a constant to the position of the string.

The truly important point is that classically thinking, *even when some strings scatter* in the way that some pieces of the incoming strings get distributed in new combinations forming the outgoing strings the system of vectorial values taking on by the derivatives  $\dot{X}_{l'}^{\iota\mu}(\tau_{l'} - \sigma_{l'})$  for the various strings  $l'$  does *not* have time to *change* in the in principle infinitely short moments of “time” when some of the

strings just touch each other. The conclusion from these remarks is the theorem(s) about the “images” of the derivatives  $\dot{X}_R^{\iota\mu}$  and  $\dot{X}_L^{\iota\mu}$  in the Minkowski space of 25 +1 dimension(s) saying that these “images” are preserved in time. [1–3].

The crux of the matter is that it is as if there are some “objects” corresponding to any vectorial value that is taken on by these  $\dot{X}_R^{\iota\mu}$  or  $\dot{X}_L^{\iota\mu}$  and such “objects” do not appear or disappear neither as time goes on while the isolated strings just develop, nor while the strings even scatter classically thinking.

This then opens up the possibility of describing the *whole situation* - the whole state of the multistring system - by means of these “objects”. That is to say: we let the state of the system of strings - to be a string field theory “Fock”-like state - be described by these “objects” meaning really the vectorial values taken on by the derivative variables  $\dot{X}_R^{\iota\mu}$  and  $\dot{X}_L^{\iota\mu}$ . If we do so, you should see, that we obtain that the state of the “objects” will not change according to the introductory discussion - at least up to the problems with the additive constants in e.g. the string positions.

This opens the possibility of describing the string field theory “Fock”-like state by the in time not developing “objects”. That is to say that we have now found a way to describe the “Fock”-like states as *non-moving*.

## B. Discretization and Even / Odd Story

In order to get a more precise formalism we must imagine to *discretize* the variables  $\tau_{Rl} = \tau_l - \sigma_l$  and  $\tau_{Ll} = \tau_l + \sigma_l$  into a series of discrete points. There shall of course be some parameter  $a$  which shall go to zero, and then in that limit  $a \rightarrow 0$  the continuum variables should be effectively reestablished. We should also have in mind that these variables are only well defined, when the “gauge” of single string theory has been fixed. This gauge means that even after the gauge fixing that were already made before we even got to the formulation with d’Alembertian equation of

motion by replacing say the Nambu action by the quadratic form  $\int[(\partial_\tau X^\mu)^2 - (\partial_\sigma X^\mu)^2]d\sigma d\tau$ , there still *remains some reparametrization* of the variables  $(\tau, \sigma)$  as being allowed (as “gauge” symmetry of the theory). In fact this left over or remaining reparametrization - after the first gauge choice - is especially simple, if expressed in terms of the “mover”-variables  $(\tau_R, \tau_L) = (\tau - \sigma, \tau + \sigma)$ , in which formulation we obtain a new set of allowed variables  $(\tau'(\tau, \sigma), \sigma'(\tau, \sigma))$  very simply, when we talk about

$$\tau'_R = \tau' - \sigma' \quad (4)$$

$$\tau'_L = \tau' + \sigma', \quad (5)$$

namely as a transformation

$$\tau'_R = \tau'_R(\tau_R) \quad (6)$$

$$\tau'_L = \tau'_L(\tau_L). \quad (7)$$

We shall in the present article make use of what is called “light cone gauge” and consists in using infinite momentum frame, i.e. the metric tensor

$$g_{\mu\nu} = \left( \begin{array}{cc} + & - \\ \begin{matrix} 0 & 1 & 0 & \cdots & \cdots & 0 \\ 1 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & -1 & \cdots & \cdots & 0 \\ \vdots & \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & \cdots & -1 & 0 \\ 0 & 0 & \cdots & \cdots & 0 & -1 \end{matrix} \end{array} \right) \left. \begin{array}{c} + \\ - \\ \\ \\ \\ \end{array} \right\} 24 \quad (8)$$

We can - and this is the “light-cone gauge” - imagine that we as a gauge choice insist on a certain fixed value for  $\dot{X}_{\iota R}^+(\tau_{\iota R})$  (and also for the left  $\dot{X}_{\iota L}^+(\tau_{\iota L})$  in closed string case).

Now also remember that as result of the gauge invariance of the Nambu action

$$S \propto \int \sqrt{(X' \cdot \dot{X})^2 - \dot{X}^2 * X'^2} d\sigma d\tau \quad (9)$$

there appear *constraints* concerning the derivatives  $X'$  and  $\dot{X}$  of the 26-vector fields on the string  $X = X^\mu$ . These constraints are well-known to be

$$X'^2 + \dot{X}^2 = 0, \quad (10)$$

$$X' \cdot \dot{X} = 0, \quad (11)$$

which by trivial algebra written into the  $\dot{X}_R = \frac{1}{2}(\dot{X} - X')$  and  $\dot{X}_L = \frac{1}{2}(\dot{X} + X')$  notation becomes

$$\dot{X}_R^2 = 0, \quad (12)$$

$$\dot{X}_L^2 = 0. \quad (13)$$

These equations concern in a simple way just our variables - the “mover”-variables - differentiated, and thus they lead to simple the rule, that the “objects” lie on the light cone (in 25+1 space time). This restriction removes one dimension as degrees of freedom for the objects. Since further as we saw the +component is fixed by gauge choice, we have out of the original  $26 = 25 + 1$  variables for each point of  $\dot{X}_R$  (or for  $\dot{X}_L$ ) or equivalently out of the “objects”  $J$  right or left *only*  $26 - 1 - 1 = 24$  *left over as truly independent variables*.

We define the “objects” by putting

$$J_L^\mu(\iota, I) = \int_{\tau_{L\iota}(I-\frac{1}{2})}^{\tau_{L\iota}(I+\frac{1}{2})} \dot{X}_L^\mu(\tau_L) d\tau_L, \quad (14)$$

$$J_R^\mu(\iota, I) = \int_{\tau_{L\iota}(I-\frac{1}{2})}^{\tau_{L\iota}(I+\frac{1}{2})} \dot{X}_L^\mu(\tau_L) d\tau_L, \quad (15)$$

and letting it be understood that the “separation” values such as  $\tau_{L\iota}(I - \frac{1}{2})$  and  $\tau_{L\iota}(I + \frac{1}{2})$  and the analogous ones for  $R$  are to be constructed so that the +components indeed become simple constants proportional to the discretization parameter  $a$ .

The restrictions on the  $\dot{X}_R$  and  $\dot{X}_L$  of course leads to the corresponding ones for the “object”-variables  $J_R$  and  $J_L$  (for small  $a$  of course):

$$J_R^\mu(\iota, I) J_R^\nu(\iota, I) g_{\mu\nu} = 0, \quad (16)$$

$$J_L^\mu(\iota, I) J_L^\nu(\iota, I) g_{\mu\nu} = 0, \quad (17)$$

$$J_R^+(\iota, I) = \frac{a\alpha'}{2}, \quad (18)$$

$$J_L^+(\iota, I) = \frac{a\alpha'}{2}. \quad (19)$$

The only components, which are to be considered independent dynamical variables, are the 24 “transverse” components for which we denote the values of  $\mu$  by  $i$ , where then  $i = 1, 2, \dots, 23, 24$ .

These remaining variables  $J_R^i(\iota, I)$  and  $J_L^i(\iota, I)$  for the “objects” may at first look like, that we could take them to be independent degrees of freedom, but the non-zero commutation rules (3) does not allow to have the different  $J_R$ -variables commute among each other. It were a major progress in the development of our string field theory project to find an idea of how to make the commutation rule (3) get consistent with a discretization formulation. The trick is to *let only every second - say the ones with even  $I$  - of the  $J_R^i(\iota, I)$  and the  $J_L^i(\iota, I)$  be physical independent degrees of freedom*. Then the “odd”  $I$  numbered  $J_R^i(\iota, I)$  and  $J_L^i(\iota, I)$  shall instead be *constructions made from the conjugate variables of the even ones*. In fact we shall take

$$J^i(I) = -\pi\alpha' \{ \Pi^i(I+1) - \Pi^i(I-1) \}, \quad (20)$$

where  $\Pi^i(I)$  is the conjugate variables to  $J^i(I)$  - we shall of course imagine to put on either  $R$  or  $L$  in the closed string case. That is to say we have the commutation rule

$$[J^i(I), \Pi^k(K)] = i\delta^{ik}\delta_{IK} \quad (21)$$

(where  $K$  is an enumeration index of the same type as  $I$  and the index  $R$  or  $L$  in the only open string case as well as the string enumerating  $\iota$  have been left out for simplicity) which we a priori now only assume for even  $I$  and  $K$  in the philosophy that it is *only the even numbered “objects” that truly exist* and thus shall be considered separate degrees of freedom. Really we only use the notation of  $\Pi^i(I)$  for *even  $I$* .

Notice that we have now come to describe a system of *arbitrarily* many strings by means of two (it turns out that for theories with also open strings only a union of the right and the left objects shall be used instead of the two classes of objects for the only closed case.) sets of “even objects” - meaning “objects” with even number enumerations - each having 24 variables  $J_R^i$  or  $J_L^i$  and their canonically conjugate variables  $\Pi_R^i$  and  $\Pi_L^i$ .

If we ignore quantum fluctuations of the “objects”, and if they come from continuous strings the objects coming e.g. from the right-mover  $\dot{X}_R^\mu$  will lie on a continuous closed curve, we can call it



a “cyclically ordered chain”. In fact the objects along such a chain is indeed ordered, so that each member - each object - has a successor. This is true whether we consider only the even objects or include the odd ones too. Because of the continuity of the chain, you could essentially - i.e. with very little/few mistakes - deduce the ordering from the values of the  $J_R^\mu$ 's, so that delivering the *information about the ordering of the chains is almost superfluous*.

A philosophical attitude like the following becomes possible because of this containment of the information about the ordering already present in the values of the  $J$ 's:

**We have the freedom to declare that the ordering is not a fundamentally existing “degree of freedom” and thus of saying: There is no ordering information in the fundamental string field theory “Fock space”, it has at the end to be extracted from the “object”  $J$  degrees of freedom alone, by using the continuity.**

If we satisfy ourselves with this observation saying, that we do not need the ordering information of the objects along the chain corresponding to a string, then we can imagine simplifying the string field theory to only have information about the - actually only the even - separate “objects”. This is a significant simplification and contributes significantly to make our string field theory very simple compared to competing string field theories.

With this simplification - of throwing away the ordering in chain information - all that has to be described in the string field theory of ours is then: How many “objects” have a given combination of its 24 degrees of freedom ( $J_R^i, \Pi_R^i$ ) (and in the closed strings only case also ( $J_L^i, \Pi_L^i$ ))? That can then be described by constructing a Fock-space of the usual particle type, in which we construct a creation and annihilation operator for each value combination of say  $J_R^i$ . That is to say that we could e.g. decide to write the creation  $a^+(J_R)$  and annihilation operators  $a(J_R)$  as depending on the  $J_R^i$ -variables taken only for the even objects. An alternative would be to write the creation  $b^+(\Pi_R)$  and annihilation operators  $b(\Pi_R)$  as functions of the  $\Pi_R^i$  variables, but as is well-known from ordinary quantum field theories you have to make a choice of one or of the other. This is analogous to, that one in quantum field theory must choose either to write the creation and annihilation operators as function of momentum  $a^+(\vec{p})$  and  $a(\vec{p})$  ( and spin) or as function of the position variables  $\phi^+(x)$  (= the second quantized field) and  $\phi(x)$ .

Analogously to the expansions of the fields  $\phi(x)$  in terms of the momentum bases  $a(\vec{p})$  in quantum field theory we must have of course an analogous relation now of the form

$$b(\Pi_R) \propto \int a(J_R) \exp(i \sum_{i=1}^{24} \Pi_R^i J_R^i) d^{24} J. \quad (22)$$

Both the  $a(J_R)$  and the  $b(\Pi)$  should act on the same Fock space, which now is in our model the Fock space for the second quantized string. We should of course in the only closed string theory case have a Fock space which is a Cartesian product  $\mathcal{H}_R \otimes \mathcal{H}_L$  of the one  $\mathcal{H}_R$  based on  $J_R$  and  $\Pi_R$  with an analogous one  $\mathcal{H}_L$  based on the left mover variables  $J_L$  and  $\Pi_L$  instead.

In the also-open-string case, however, we get the mover-sets of variables mixed up and could leave out the index  $R$  or  $L$ . This comes about because at the end of an open string the right mover wave gets reflected as a left mover one or oppositely.

One should notice that the formulation of a state of several strings in the formalism of these “objects” - or equivalently in terms of the string field theory Fock space of ours describing the number of “objects” with a given value-set for  $J_R$  say - there is no time development, because the “objects” are static according to the above mentioned theorem of the non-variation of the images of the right and left mover  $\dot{X}_R$  and  $\dot{X}_L$  functions into the 25+1 Minkowski spaces (where they even only fall on the light cones). In other words our Fock space describes the world of strings at all times. Only the translation to the string language is needed, no time development calculation.

### C. The “Object” to String Way

Let us now shortly contemplate the opposite way of describing our string field theory, namely to start from thinking on the “objects” described by their Fock space, and then look for how one should get to translate that theory into a theory of a number of strings.

So imagine that we have a “fundamental” model (which should of course, if string theory were the theory of everything(“T.O.E.”), according to our picture be the fundamental theory of nature) described by a Hilbert space or Fock space on which we have defined the operators  $a_R(J_R)$  and  $a_L(J_L)$  (and in addition one can define their Fourier transformed  $b_R(\Pi_R)$  and  $b_L(\Pi_L)$ ) operators annihilating “objects” with their 24-component  $J_R$  quantum numbers being just  $J_R = (J_R^1, J_R^2, \dots, J_R^{24})$ . In the only-closed-string model we have also corresponding  $L$ -operators, which we leave out for simplicity, or the reader may think of the open string case and ignore the indexes  $R$  ( and  $L$ ).

Notice that our “fundamental” model is really extremely *simple*: It is like a quantum field theory in which the particle has just 24 spatial dimensions and nobody cares for any development in time.

Then the story this way -i.e.*from “objects” to strings* - is a story about making mathematical definitions by means of the “objects” and then reach to *rewrite the state of the “objects” into a state of some number of strings*. The string formulation looks more complicated than our “object” formulation!

Let us here just enumerate the main steps, and leave it for the reader to grasp what shall go on by looking on the other way description above or by reading our other articles e.g. [3]:

- Interpret the Fock-space state of our “fundamental” model as a quantum theory with an arbitrary number of “objects” (this is just what one always does (analogously) in quantum field theories)
- Classify the “objects” into “cyclically ordered chains” of “objects”, so that these lie in approximately continuous closed chains. Although one could imagine working on making this step well-defined, it is a priori somewhat ambiguous - and of course depends on “objects” in states with quantum fluctuation. We must work more on this step, but the ambiguity may be important for at all getting scattering in our string theory at the end.
- Once we have these cyclically ordered chains of (fundamental even) “objects” we *invent* purely mathematically “odd objects”, which by definition in the now doubled cyclically ordered chains sit in between the original even or fundamental “objects”. The odd “objects” are of course having their  $J_R$  constructed as differences of the conjugate variables to the in the cyclically ordered chains neighboring even objects.
- (Here is again a little trouble to be worked on), but let us next pair a right and a left cyclically ordered chain and then construct at least the derivatives  $\dot{X}$  and the  $\sigma$ -derivative  $X'$  for all the combinations of a right and a left “objects” in the two chains which are being combined.
- The strings determined at least essentially from these derivatives are now the strings corresponding to the Fock state we started from.

It is to be more worked out in our model, if the lack of getting the integration constant in the position of the strings can be identified with the ambiguity of choosing the origin of the 25+1 dimensional Minkowski space, in which the strings are present.



With respect to at least the “transverse” momentum components, i.e. the ones with  $\mu = i = 1, 2, \dots, 24$ , it is easy to see that the even(or fundamental) “objects” function as constituents in the sense that the “transverse” momentum of the string is given as the sum over these even “object”  $J$ -components. The odd “objects” (the constructed ones) namely cancel out in the sum giving the momentum, because a certain  $\Pi(I)$  contributes in two odd  $J$ ’s with opposite signs, and multiplied by the same  $-\pi\alpha'$ . In this sense the even objects function much like constituents while then the string is the composed object the bound state. But it is a bit more complicated in detail.

Nevertheless we shall illustrate as an attempt to be pedagogical in the following describe an analogy to bound states and their constituents.

### III. SCATTERING BY JUST EXCHANGE IS THINKABLE

In the ontological picture of our string field theory the Hilbert space of possible state of the world of a string theory is in fact the Fock space for what we call “objects”. These “objects” are each essentially a particle or a system with 24 degrees of freedom, meaning 24  $J^i(I)$ -variables and 24  $\Pi^i(I)$ - variables canonically conjugate to the  $J^i$ ’s.

Even when these “objects” in a slightly complicated way are rewritten to describe scattering of strings, they themselves do not develop even during the scattering. In our “object” - formulation everything is totally static, or rather there is no time, it is timeless.

This scattering without anything changing sounds a priori very strange. Therefore we would like here to give at least an idea of how that strange phenomenon can come about:

Suppose that we had a couple of series of constituent particles making up some composite particles (essentially bound states, but they might not even be bound; rather just formally considered composed). Now if one suddenly decide to divide the “constituent” particles into groups forming composites in a different way from at first.

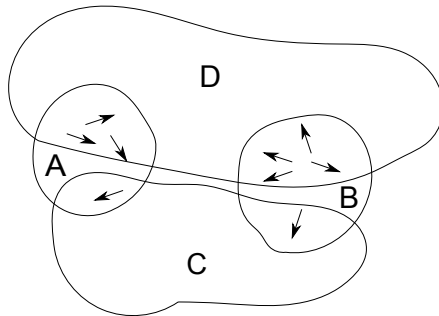


FIG. 2: We illustrate how one may look at a set of (independent) “constituents” forming first two clumps  $A$  and  $B$ , while later we divide them into two clumps  $C$  and  $D$  in a different way.

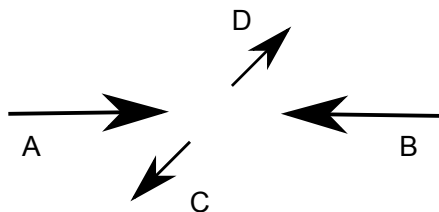


FIG. 3: Counting the momenta of the clump  $A, B, C, D$  of the foregoing figure 2 as the sum of the momenta of the “constituents” we get the picture of a scattering  $A + B \rightarrow C + D$ .

Then the momenta of the composite clumps after the considering the new ones will typically be quite different from those of the initial composite clumps.

This is a quite trivial remark: If we reclassify some constituents into a new set of classes of constituents (i.e. new composites) of course it will look like scattering of the composites.

In this way we hope that the reader can see that there is also a chance that making up a model on our “objects”: these “objects” can function much like the just mentioned “constituents” and thus we could also in our model have some pretty formal scatterings. In the way we here think of the scatterings these scatterings are something we only think upon. The constituents and analogously our “objects” do not change their momenta, say, at all. We have in this sense presented the idea of having completely formal scattering without anything going on at all for our “objects”. It is our hope in the long run to argue that in spite of this scattering being very formal we shall at the end get for it scattering amplitudes becoming the Veneziano model amplitudes.

#### IV. CORRESPONDENCE FOR SINGLE (OPEN) STRING

For a cyclically ordered chain of objects – considering only the even ones (as existing) and only the “transverse  $J^i$  and  $\Pi^i$ ” we have a wave function

$$\psi(J^i(0), J^i(2), \dots, J^i(N-2)) \quad (1)$$

(i.e. it is a wave function defined on a  $24 \cdot \frac{N}{2}$  dimensional space.)

Since such a chain represents an open string, this wave function should be essentially in correspondence with an open string state meaning a single string Fock space constructed from the creation and annihilation operators  $a_n^i$  and  $a_n^{i\dagger} = a_{-n}^i$ . In the GGRT[21] we only have the transverse components  $a_n^i, a_n^{i\dagger}$  with  $i = 1, 2, \dots, 24$ . “Essentially” here means that the  $J^i(I)$ , and  $\Pi^i(I)$  I even gives us the  $\dot{X}_R^i$  and  $\dot{X}_L^i \simeq \dot{X}_R^i$  but we need an extra discussion to identify the “average” position of the string associated with the cyclically ordered chain to be the average of  $\Pi^i(I)$ ’s (which we postpone till later). Also the permutation of the “objects” needs discussion.

Thus in principle there exists an operator  $F$  mapping a wave function state  $\psi$  for the cyclically ordered chain into the corresponding state  $|\text{str } \psi\rangle$  of the open string in the conventional open string Fock space notation:

$$|\text{str } \psi\rangle = F\psi. \quad (2)$$

One way to construct this correspondence  $F$  could be to discretize the string analogously to the discretization, we used for  $\dot{X}_R^\mu(\tau_R) = \dot{X}_L^\mu(\tau_R)$ . This means that we consider the string at a specific “time”  $\tau$  (say  $\tau = 0$ ) and the discretize  $\sigma$  in intervals so that  $\int_{\text{an interval}} X_R^\mu d\sigma = \frac{\alpha' a}{2}$ .

If we take the discretized steps in  $\sigma$  to match with the discretization used for our objects it should mean that for each little discretized interval in  $\sigma$ , say  $\Delta\sigma$  we obtain to say  $\int_{\Delta\sigma} \dot{X}^\mu d\sigma$  comes from just two (usually different) “objects” – one functioning as right-mover, the other one as left-mover -. The in  $\sigma$  discretized string variables are then of the form

$$\begin{aligned} \int_{\sigma-\Delta\sigma/2}^{\sigma+\Delta\sigma/2} X^{\mu'}(\sigma, \tau) d\sigma &= X^\mu\left(\sigma + \frac{\Delta\sigma}{2}, \tau\right) - X^\mu\left(\sigma - \frac{\Delta\sigma}{2}, \tau\right) \\ &= \int_{\sigma-\Delta\sigma/2}^{\sigma+\Delta\sigma/2} \{X_R^{\mu'}(\tau - \sigma) + X_L^{\mu'}(\tau + \sigma)\} d\sigma \\ &= -J^\mu(I) + J^\mu(K) \end{aligned} \quad (3)$$

and

$$\begin{aligned} \int_{\sigma-\Delta\sigma/2}^{\sigma+\Delta\sigma/2} \dot{X}^\mu(\sigma, \tau) d\sigma &= \int_{\sigma-\Delta\sigma/2}^{\sigma+\Delta\sigma/2} \pi \alpha' \Pi^\mu(\sigma, \tau) d\sigma \\ &= J^\mu(I) + J^\mu(K) \end{aligned} \quad (4)$$

where now if we assume say  $I \simeq 0$  corresponds to  $\sigma = 0$  we have

$$I \simeq \frac{\sigma}{2\pi/N} = \frac{N\sigma}{2\pi}, \quad (5)$$

$$K \simeq -\frac{\sigma}{2\pi/N} = -\frac{N\sigma}{2\pi}. \quad (6)$$

Really we should not write our identification relations (3,4) but rather we should include into them the operator  $F$  going from the chain-Hilbert space to the string-one so that the two sides of the equations (3,4) act on the same Hilbert space:

$$\begin{aligned} F^{-1} \int_{\sigma-\Delta\sigma/2}^{\sigma+\Delta\sigma/2} X^{\mu'}(\sigma, \tau) d\sigma F &= F^{-1} \left\{ X^\mu \left( \sigma + \frac{\Delta\sigma}{2}, \tau \right) - X^\mu \left( \sigma - \frac{\Delta\sigma}{2}, \tau \right) \right\} F \\ &= F^{-1} \int_{\sigma-\Delta\sigma/2}^{\sigma+\Delta\sigma/2} \{ X^{\mu'}(\tau - \sigma) + X^{\mu'}(\tau + \sigma) \} d\sigma F \\ &= -J^\mu(I) + J^\mu(K) \\ &= -J^\mu \left( \frac{N\sigma}{2\pi} \right) + J^\mu \left( -\frac{N\sigma}{2\pi} \right). \end{aligned} \quad (7)$$

## V. FOURIER TRANSFORMING

We shall now seek the precise correspondence between the well-known  $\alpha_n^\mu$  annihilation and for negative creation operators given in the notation of the formula (2.1.56) in the book by Green, Schwarz and Witten [18]

$$X^\mu(\sigma, \tau) = x^\mu + l^2 p^\mu \tau + il \sum_{n \neq 0} \frac{e^{-in\tau}}{n} \alpha_n^\mu \cos(n\sigma) \quad (1)$$

and the  $c_L^\mu$  and  $d_L^\mu$  in our previous article [1]

$$c_L^i = \frac{1}{N} \sum_{\substack{I=0,2,4,\dots \\ I \text{ even}}}^{N-2} J^i(I) e^{-\frac{i2\pi L \cdot I}{N}} \quad (2)$$

and

$$d_L^i = \frac{1}{N} \sum_{\substack{I=0,2,4,\dots \\ I \text{ even}}}^{N-2} \Pi^i(I) e^{-\frac{i2\pi L \cdot I}{N}}. \quad (3)$$

We can instead of these last two equations defining  $c_L^i$  and  $d_L^i$ , namely (53-54), use the opposite Fourier transformations

$$\begin{aligned} J^i(I) &= Re \left\{ \sum_{L=0}^{N-1} c_L^i \exp \left( \frac{iL \cdot I 2\pi}{N} \right) \right\} \\ &= 2Re \left\{ \sum_{L=0}^{\frac{N}{2}-1} c_L^i \exp \left( \frac{iL \cdot I 2\pi}{N} \right) \right\} \end{aligned} \quad (4)$$

and (55) in [1]

$$\begin{aligned} \Pi^i(I) &= Re \left\{ \sum_{L=0}^{N-1} d_L^i \exp \left( \frac{iL \cdot I 2\pi}{N} \right) \right\} \\ &= Re \left\{ \sum_{L=0}^{\frac{N}{2}-1} d_L^i \exp \left( \frac{iL \cdot I 2\pi}{N} \right) \right\} \end{aligned} \quad (5)$$

for even  $I = 0, 2, \dots, N-4, N-2$  and  $i = 1, 2, 3, \dots, 24$ . Here the  $c_L^i$ 's and  $d_L^i$ 's obey

$$c_L^i = c_{L+\frac{N}{2}}^i = (c_{-L}^i)^* \quad (6)$$

and

$$d_L^i = d_{L+\frac{N}{2}}^i = (d_{-L}^i)^*. \quad (7)$$

In (1) the quantity

$$l = \sqrt{2\alpha'} = \frac{1}{\sqrt{\pi T}} \quad (8)$$

were just introduced to provide a length scale. Here  $T$  is the string tension and  $\alpha'$  the Regge slope.

We may differentiate (1) with respect to  $\tau$  and insert the  $l$ -value  $\sqrt{2\alpha'}$  to obtain

$$\dot{X}^\mu(\sigma, \tau) = 2\alpha' p^\mu + \sqrt{2\alpha'} \sum_{\substack{n \neq 0 \\ -\infty}}^{\infty} e^{-in\tau} \alpha_n^\mu \cos(n\sigma). \quad (9)$$

Similarly we could differentiate with respect to  $\sigma$  to obtain

$$X^{\mu'}(\sigma, \tau) = -i\sqrt{2\alpha'} \sum_{\substack{n \neq 0 \\ -\infty}}^{\infty} e^{-in\tau} \alpha_n^\mu \sin(n\sigma). \quad (10)$$

Remembering that locally

$$X^\mu(\sigma, \tau) = X_R^\mu(\tau - \sigma) + X_L^\mu(\tau + \sigma) \quad (11)$$

so that

$$\dot{X}^\mu(\sigma, \tau) = \dot{X}_R^\mu(\tau - \sigma) + \dot{X}_L^\mu(\tau + \sigma) \quad (12)$$

and

$$X^{\mu'}(\sigma, \tau) = -\dot{X}_R^\mu(\tau - \sigma) + \dot{X}_L^\mu(\tau + \sigma) \quad (13)$$

we obtain easily

$$\dot{X}_L^\mu(\tau + \sigma) = \frac{1}{2} \left\{ \dot{X}^\mu(\sigma, \tau) + X^{\mu'}(\sigma, \tau) \right\} \quad (14)$$

and

$$\dot{X}_R^\mu(\tau - \sigma) = \frac{1}{2} \left\{ \dot{X}^\mu(\sigma, \tau) - X^{\mu'}(\sigma, \tau) \right\}. \quad (15)$$

Inserting (9,10) into these last two equations (14,15) we obtain:

$$\begin{aligned} \dot{X}_L^\mu(\tau + \sigma) &= \frac{1}{2} \left\{ \dot{X}^\mu(\sigma, \tau) + X^{\mu'}(\sigma, \tau) \right\} \\ &= \sqrt{2\alpha'} \sum_{\substack{n \neq 0 \\ n=-\infty}}^{+\infty} \alpha_n^\mu \{ \cos(n\sigma) - i \sin(n\sigma) \} e^{-in\tau} + \alpha' p^\mu \\ &= \alpha' p^\mu + \sqrt{2\alpha'} \sum_{\substack{n \neq 0 \\ n=-\infty}}^{+\infty} \alpha_n^\mu e^{-in\tau - in\sigma} \\ &= \alpha' p^\mu + \sqrt{2\alpha'} \sum_{\substack{n \neq 0 \\ n=-\infty}}^{+\infty} \alpha_n^\mu e^{-in(\tau + \sigma)} \end{aligned} \quad (16)$$

and

$$\begin{aligned} \dot{X}_R^\mu(\tau - \sigma) &= \frac{1}{2} \left\{ \dot{X}^\mu(\sigma, \tau) - X^{\mu'}(\sigma, \tau) \right\} \\ &= \alpha' p^\mu + \sqrt{2\alpha'} \sum_{\substack{n \neq 0 \\ n=-\infty}}^{+\infty} \alpha_n^\mu e^{-in(\tau - \sigma)}. \end{aligned} \quad (17)$$

If we put  $\tau = 0$  and discretize using  $I = \frac{\sigma N}{2\pi}$  we may identify say

$$e^{-in(\tau - \sigma)} = e^{+i\frac{2\pi IL}{N}} \quad (18)$$

for the  $\dot{X}_R$ , where we took

$$n = L \quad (19)$$

and for the  $\dot{X}_L$

$$e^{-in(\tau + \sigma)} = e^{i\frac{2\pi IL}{N}} \quad (20)$$

also with  $n = L$ . Then we have of course

$$\int \cdots d\sigma \Leftrightarrow \sum_{I \text{ even}} \cdots \frac{2\pi}{N} \cdot 2.$$

(the last factor 2 from  $I$  even only)

In this way we see comparing to

$$\begin{aligned}
J^i(I) &= \int_{\tau_R(I-\frac{1}{2})}^{\tau_R(I+\frac{1}{2})} \dot{X}_R^i(\tau_R) d\sigma \\
&= \frac{2\pi}{N} \cdot \dot{X}_R^i(\tau_R(I)) \\
&= 2Re \left\{ \sum_{L=0}^{\frac{N}{2}-1} c_L^i \exp\left(\frac{iL \cdot I 2\pi}{N}\right) \right\}
\end{aligned} \tag{21}$$

that

$$\alpha' p^i + \sqrt{2\alpha'} \sum_{\substack{n \neq 0 \\ n=-\infty}}^{+\infty} \alpha_n^\mu e^{-in(\tau-\sigma)} = \frac{N}{2\pi} * 2Re \left\{ \sum_{L=0}^{\frac{N}{2}-1} c_L^i \exp\left(\frac{iL \cdot I 2\pi}{N}\right) \right\}. \tag{22}$$

Now we use the identifications (18) and comparing the coefficients we obtain first for  $n = L \neq 0$

$$\sqrt{2\alpha'}(\alpha_n^\mu + \alpha_{-n}^\mu) = \frac{N}{2\pi}(c_n^\mu + c_n^{\mu\dagger}). \tag{23}$$

By nearer contemplation indeed we can obtain

$$\alpha_n^\mu = \frac{N}{2\pi\sqrt{2\alpha'}} c_n^\mu \tag{24}$$

This is true for smoothly lying strings because we “swindled” a bit by only using the even  $I$ .

We could instead have used the odd  $I$ 's still assuming smoothness and then we can express the relation between the  $d_L^i = d_n^i$ 's of the cyclically ordered chain and the  $\alpha_n^i$ 's. Let us in fact first insert (5) into the formula

$$J^i(I) = -\pi\alpha' \{ \Pi^i(I+1) - \Pi^i(I-1) \} \tag{32}$$

from [1] for the odd  $I$  “objects” so as to obtain

$$J^i(I) = -2\pi\alpha' Re \left[ \sum_{L=0}^{\frac{N}{2}-1} d_L^i \left\{ \exp\left(\frac{iL(I+1)2\pi}{N}\right) - \exp\left(\frac{iL(I-1)2\pi}{N}\right) \right\} \right] \tag{33}$$

which in turn like just above is

$$\begin{aligned}
J^i(I) &\simeq \frac{2\pi}{N} \dot{X}_R^i \left( \frac{I 2\pi}{N} \right) \\
&\simeq -\alpha' Re \left( 2i \sum_{n=1}^{\infty} n d_n^i e^{\frac{inI \cdot 2\pi}{N}} \right).
\end{aligned} \tag{34}$$

Using  $\alpha_{-n}^i = \alpha_n^{i\dagger}$  we can write the left hand side as

$$\alpha' p^i + \sqrt{2\alpha'} 2Re \left( \sum_{n=1}^{\infty} \alpha_n^i e^{-in(\tau-\sigma)} \right) \tag{35}$$



and we deduce

$$-\alpha' 2i n d_n^i = \sqrt{2\alpha'} 2\alpha_n^i \quad (36)$$

using the identification (18):

$$e^{-in(\tau-\sigma)} = e^{i\frac{2\pi I n}{N}}. \quad (37)$$

So we obtain by comparison:

$$\begin{aligned} \alpha' p^i + \sqrt{2\alpha'} \sum_{\substack{n \neq 0 \\ n=-\infty}}^{+\infty} \alpha_n^i e^{-in(\tau-\sigma)} \\ = -2\pi\alpha' \cdot \frac{N}{2\pi} \text{Re} \left[ \sum_{n=1}^{\frac{N}{2}-1} d_n^i \left\{ \exp\left(\frac{in(I+1) \cdot 2\pi}{N}\right) - \exp\left(\frac{in(I-1) \cdot 2\pi}{N}\right) \right\} \right] \\ \simeq -\alpha' N \text{Re} \left[ 2i \sum_{n=1}^{\frac{N}{2}-1} d_n^i \sin\left(\frac{n}{N} 2\pi\right) e^{\frac{inI 2\pi}{N}} \right]. \end{aligned} \quad (38)$$

Simplifying we get

$$d_n^i = +i \sqrt{\frac{2}{\alpha'}} \frac{\alpha_n^i}{n}. \quad (39)$$

In [3] as formula (90) there had a smoothness requirement

$$c_L^i \simeq -i 4\pi \cdot \frac{\alpha'}{N} d_L^i L \quad (40)$$

which with  $n = L$  reads

$$c_n^i \simeq -i \frac{4\pi\alpha'}{N} d_n^i n. \quad (41)$$

It is a self-consistency check that our two equations (24) and (39)

$$c_n^i = \frac{2\pi\sqrt{2\alpha'}}{N} \alpha_n^i \quad (42)$$

$$d_n^i = i \sqrt{\frac{2}{\alpha'}} \frac{\alpha_n^i}{n} \quad (43)$$

leading to

$$c_n^i = \frac{i 2\pi\alpha' n}{N} d_n^i \quad (44)$$

(which apart from a factor 2 is the continuity condition). We should – if we consider the Hilbert spaces of the chain of objects and the string as different – really not have written our identification equations as we just did but rather have included an  $F$  so as to rather be e.g.

$$c_n^i = \frac{2\pi\sqrt{2\alpha'}}{N} F^{-1} \alpha_n^i F \quad (45)$$

and

$$d_n^i = i \sqrt{\frac{2}{\alpha'}} F^{-1} \frac{\alpha_n^i}{n} F. \quad (46)$$

## VI. STRING UNIFICATION

Now the main point of our string field theory in terms of the (actually even) “objects” is that e.g. a splitting of an open string into two open strings in our model just reflects that one cyclically ordered chain “after the decay” get interpreted as two such chains. That is to say “ontologically” nothing happens, it is only that we “after the decay” think of two chains. Now we shall make the assumption that even though there are big quantum fluctuations there is still some element of continuity in the string and thus also in our cyclically ordered chains, so that the variation with  $I$ , odd or even, of  $J^\mu(I)$  is at least somewhat continuous (in a crude way). Such a continuity if it is implemented statistically in the type of cyclically ordered chains which we consider, will mean that the probability for two different pictures of a system of objects in terms of chains will be more likely to be able to describe some object-situation the more the hanging together in the two chain systems are the same. Let us attempt to say that in more detail: A certain set of even “objects” can be arranged into chains in many ways of course. See for instance on the figure here of say 18 even “objects” can be put on cyclically ordered chains like this.

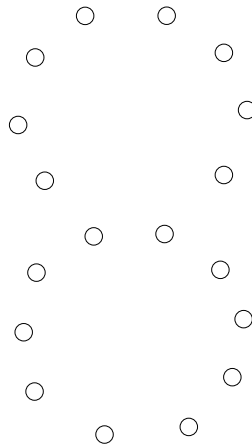


FIG.4

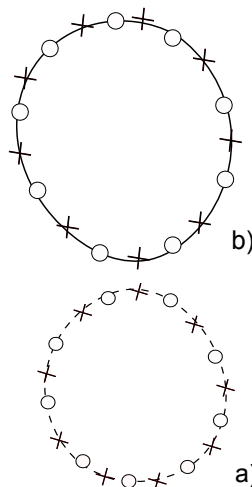


FIG.5

It is put on two chains. We can also put the same even objects, just on a single cyclically ordered

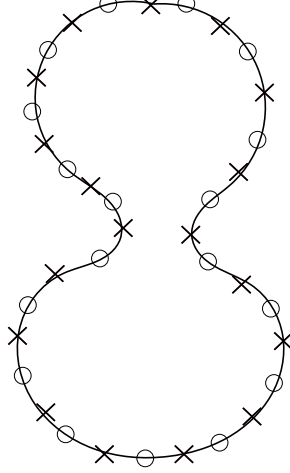


FIG.6

chain like this.

Now we could in principle take the “objects” in any order along the chains, but we shall by the “assumption of continuity” assume that the state of the even objects “found in nature” is so that it may only be a few systems of chains of specific ordering that matches to have successive objects close in  $J^i$  and  $\Pi^i$  spaces to their neighbors in the cyclically ordered chain in which they sit. If we denote the “objects” number  $I$  in the chain number  $\iota$  as  $J^i(I, \iota)$  the  $\Pi^i$  as  $\Pi^i(I, \iota)$  we mean by continuity

$$J^i(I, \iota) \simeq J^i(I \pm 2, \iota), \quad (47)$$

$$\Pi^i(I, \iota) \simeq \Pi^i(I \pm 2, \iota). \quad (48)$$

Really we want even to assume such continuity even for odd “objects” on the chains so that we have

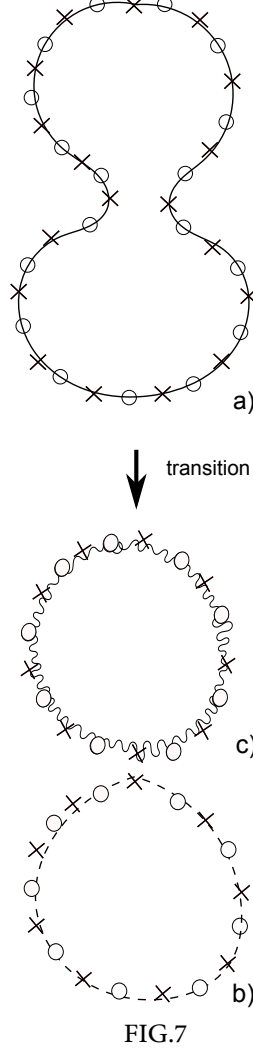
$$J^\mu(I, j) \simeq J^\mu(I \pm 1, j). \quad (49)$$

With such continuity the probability that two different sets/systems of cyclically ordered chains of objects can match – with continuity – to the some sets of “objects” is of course biggest if most neighboring objects with respect to one set of chains are also neighbors with respect to the other one. This means that if e.g. a single cyclically ordered chain of objects under a “decay” shall go to be in thought replaced by say two cyclically ordered chains the continuity assumption makes it have highest probability to match the continuity if as many object-neighbors remain neighbors after the rethinking of the way the objects are put into chains. To make by rethinking one circular chain into two one needs topologically to violate the neighborship at two places at last on the single-starting chain. But since probability of matching with continuity is higher the lower the number of neighborship violations we suggest that in first approximation we shall think of rethinkings into a new chain-system which violate minimally the neighborship of the even “objects”.

Especially we shall think of the dominant rethought or “decayed” single cyclically ordered chain into two chains as a change in the chaining consisting in cutting the initial chain at two places and choosing the two pieces to the two new chains. This is illustrated by the transition:

To be understood that full line  $\text{---}\circ\text{---}\circ\text{---}\circ\text{---}\circ\text{---}\circ\text{---}$  chain gets split into the  $\text{---}\circ\text{---}\circ\text{---}\circ\text{---}$  and the  $\sim\circ\sim\circ\sim\circ\sim\circ\sim\circ\sim$  chains. This is done by only breaking the full curve  $\text{---}\circ\text{---}\circ\text{---}\circ\text{---}\circ\text{---}$  at two places.

The basic idea to obtain scattering amplitudes or vertices in our SFT consists in:



- 1) Put up the state of the in general several strings (but in the case a string decay only one string) in the initial state and compute their state to be described in terms of “objects” (really even objects at the end).

The reformulation from one string to one cyclically ordered chain is done by the formulas above. The composition of several strings  $j = 1, 2, \dots, m$  to a SFT state with  $m$  strings is in our picture simply done by considering the union of the sets of even objects associated with the strings in the initial state.

It becomes a state with

$$\frac{1}{2} (N_1 + N_2 + \dots + N_\iota + \dots + N_m) \quad (50)$$

even objects if there are  $\frac{1}{2}N_\iota$  even objects in to the  $\iota$ -th string corresponding cyclically ordered chain.

- 2) One then considers a similar final state with some other number  $m_f$  of strings in it. From that one constructs quite analogously to the just described initial state a state consisting of  $m_f$  chains of “objects”.

- 3) One takes the quantum mechanical (Hilbert inner product) overlap of the initial state even “object” wave function  $\psi_i$  and the final state even object wave function  $\psi_f$

$$\int \psi_f^*(J) \psi_i(J) \prod_{I,\iota} \prod_{i=1}^{24} dJ^i(I, \iota). \quad (51)$$

For this overlap to make sense it is crucial that there is a (natural) matching between the even objects in the initial state with

$$J^i(I_\iota, \iota) \quad (52)$$

where  $J = 1, 2, \dots, m(=m_i)$  and  $I_\iota = 0, 2, \dots, N_\iota - 2$  and in the final state where we have  $J^i(I_\iota, \iota)$  but now with  $\iota = 1, 2, \dots, m_f$  and  $I_\iota = 0, 2, \dots, N_\iota^{(f)} - 2$ . Especially it is of course for common overlap (51) to be meaningful at least needed that total number of even objects in the initial state and in the final state is the same, i.e.

$$N_1^{(i)} + N_2^{(i)} + \dots + N_\iota^{(i)} + \dots + N_{m_i}^{(i)} = N_1^{(f)} + N_2^{(f)} + \dots + N_\iota^{(f)} + \dots + N_{m_f}^{(f)}. \quad (53)$$

In the case we use the “gauge condition” in IMF (=infinite momentum frame) that

$$J^+(I_j, \iota) = \frac{a\alpha'}{2} \quad (\text{for both even and odd } I_\iota) \quad (54)$$

for all the objects the equality of the numbers of even objects follow from the conservation of  $p^+$  (= the longitudinal momentum) and the fact that there is “all the time (i.e. in all chainings) the same numbers of even and odd “objects”. So the conservation of the longitudinal momentum  $p^+ \propto \sum_{\text{all the objects}} J^+$  ensures that the number of objects in  $\psi_i$  and  $\psi_f$  match.

### A. The trick of taking One String with Short Chain

In principle the reader should now that one should now calculate the vertex of the decay of one string into two by evaluating the overlap of two states for a certain number of even objects (proportional to  $p^+$ ) calculated respectively from the single cyclically ordered chain of the to decay open string and from the unification/product of the two cyclically ordered chains corresponding to decay-product strings.

Now it is our hope to derive that vertex coming out of such an overlap calculation between a two string state and a one-string state should be calculated to be the three string vertex.

Now, however, the technically simplest vertex presented in string theory is rather the vertex for a string in an arbitrary state emitting a tachyon (ground state) string and becoming some to be prescribed state of the string. In such a vertex we can in principle express the vertex by a vertex operator such as (2.2.59) in [18]

$$V(k, \tau) =: e^{ik \cdot X(0, \tau)} :. \quad (55)$$

Notice that such a nice vertex operator is at least without the normal ordering  $:\cdots:$  local on the string. It consists alone of string operators at the point  $\sigma = 0$ .

In our picture wherein a priori a finite part of the cyclically ordered chain breaks off – and therefore also a finite nonzero part of the string – it would be very strange physically, if that could be described by an in  $\sigma$  localized vertex operator. We would rather expect a non-local operator in  $\sigma$  with an extent of the order of the string piece breaking off. That means that, if we insist on going for a vertex with an in  $\sigma$  local vertex operator, then intuitively we would expect that the string piece breaking off or the piece of cyclically ordered chain breaking off should be so short that its length say in  $\sigma$  goes to zero.

Let us take this way of thinking and this type of vertex operator, to be the one we hope to achieve, as the suggestion to seek to make the breaking off piece infinitely small.

Such an infinitely small cyclically ordered chain of “objects” means also that its plus-component  $p^+$  is very small and thus in our “gauge”( ) having fixed to a small quantity  $\propto a$  the number of “objects” in the “small” piece is *relatively* very small (but it can still be very large in absolute number).

If one or more of the say transverse components for separating off string piece is finite, then these components will be relatively very big for the “objects” in this piece compared to the ones in the rest of the cyclically ordered chains. This in turn means that in the continuum formulation as a density in  $\sigma$  there is a delta-function contribution from the external string represented by the “small piece”.

The achievement of this special choice of arranging the “small piece” to have small plus component is that we can use *the same parametrization for the two other external strings in the vertex under construction*. This is very important for obtaining a formalism like the one used in the usual formulation of the vertex operator, in which the “small piece” gets represented by an operator, while the two other external strings are represented by respectively the ket and the bra.

Looking at a not infinitesimal scale in the  $\sigma$  the breaking off piece is infinitesimally short. Nevertheless a finite amount of say  $p^i$  momentum is added to the incoming string in order to bring it to the “outgoing one” after the “absorption ” of the counted as short string. This addition only comes in in the very small  $\sigma$  region where the “short string” gets included. To describe the insertion of the “short string” (or short cyclically ordered piece of chain of “objects”) in terms of an operator we must then have an operator that changes the momentum by the amount  $k^\mu$  - the momentum of the string related to the short cyclically ordered chain. Such an operator is precisely

$$\exp(ik \cdot X(0, \tau)). \quad (56)$$

Apart from an overall constant factor and apart from derivatives of delta-functions this is just the usual vertex. So in this sense we can claim that we derived the usual vertex in our case considered.

Our derivation was a bit quick at least with respect to that we used that we could choose the frame we wanted but assuming that the theory of ours is Lorentz invariance. That we hope but it is actually highly nontrivial.

## VII. CONCLUSION AND RESUME

### Acknowledgments

The authors acknowledge K. Murakami for informing them some useful information of the references on string field theory. One of us (H. B. N.) thanks the Niels Bohr Institute for allowance to work as emeritus and for Masao to visit to give a talk in connection with the “Holger Fest”. Masao Ninomiya acknowledges the Niels Bohr Institute and the Niels Bohr International Academy



for inviting to the “Holger Fest”. He also acknowledges that the present research is supported in part by the J.S.P.S. Grant-in-Aid for Scientific Research Nos.21540290, 23540332 and 24540293. Also H. B. N. thanks to the Bled conference participants organizers and Matijaz Breskov for financial support to come there where many of the ideas of this work got tested again.

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